Partially Dielectric-Slab-Filled Waveguide Phase Shifter

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Abstract—**An equivalent circuit of a waveguide junction between two asymmetrically filled waveguides is obtained. This equivalent circuit is applied to the design of single-section impedance-matching transformers for a dielectric-slab-filled waveguide phase shifter. Calculation and measurement indicated that when a thin alumina slab is employed, a 360° phase shift can be accomplished in a section approximately 1.1 times the unloaded waveguide wavelength at 2.7 GHz, and that the VSWR between 2.7 and 3.0 GHz is kept to less than 1.15.**

I. INTRODUCTION

RECTANGULAR waveguides asymmetrically filled with one or more dielectric slabs have long been used as phase shifters. Although the propagation of electromagnetic waves in these waveguides and some of the phase-shifter characteristics have been studied [1]–[11], [14], the basic problem of impedance matching between this asymmetrically dielectric-slab-filled waveguide and the unloaded waveguide has not been systematically attacked. Consequently, waveguide phase shifters are mostly designed with materials of small relative dielectric constant ϵ_r and with long tapered ends.

Recently, the electromagnetic-wave propagation across junctions between two waveguides symmetrically filled with E-plane dielectric slab was investigated [12]. The results of this investigation were converted into parameters (normalized impedances and admittances) of an equivalent circuit so that a systematic study of these junctions could be conducted. The understanding of these junctions led to the successful design of partially dielectric-filled impedance transformers for a microwave discharge chamber [13]. The many similarities between the symmetrically and asymmetrically filled waveguide junction suggest that all the techniques previously applied to the former can be adapted to the latter.

In this paper we present a study of various phase-shifter characteristics and the design of an impedance-matching transformer of an asymmetrically dielectric-slab-filled waveguide phase shifter.

II. TE_{0m} MODES IN AN ASYMMETRICALLY DIELECTRIC-SLAB-FILLED WAVEGUIDE

For TE_{0m} modes in either waveguide shown in Fig. 1(a), the electric field is

$$E_x = A_m F_m(y) \exp(\mp jh_m z) \tag{1}$$

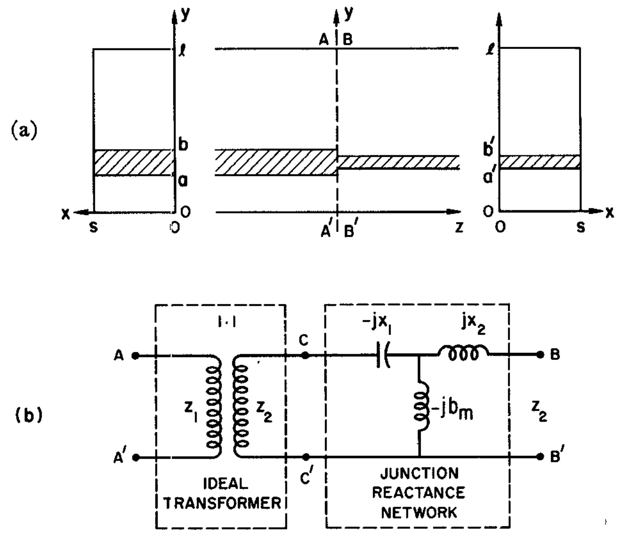

Fig. 1. Asymmetrically dielectric-slab-filled waveguide junction. (a) Geometry. (b) Equivalent circuit.

and the magnetic fields are

$$H_y = \pm E_x / Z_m \tag{2}$$

and

$$H_z = \pm j \frac{A_m}{k_0 \eta_0} \frac{dF_m(y)}{dy} \exp(\mp jh_m z) \tag{3}$$

where the upper and lower signs apply to waves propagating to the right and to the left, respectively, and h_m is the propagation constant of the mth mode. The field distribution function is given by

$$F_m(y) = \frac{1}{(\Lambda_m)^{1/2}} \cdot \begin{cases} \sin \gamma_m(l-b) \sin(a_m a + \psi_m) \sin \gamma_m y, & 0 < y < a \\ \sin \gamma_m(l-b) \sin \gamma_m a \sin(a_m y + \psi_m), & a < y < b \\ \sin \gamma_m a \sin(a_m b + \psi_m) \sin \gamma_m(l-y), & b < y < l, \ m = 1,2,3,\cdots \end{cases} \tag{4}$$

and the characteristic wave impedance is defined as

$$Z_m = k_0 \eta_0 / h_m \tag{5}$$

where

$$\eta_0 = (\mu_0/\epsilon_0)^{1/2} \quad \text{and} \quad k_0 = 2\pi f/c \tag{6}$$

where μ_0, ϵ_0, and c are the permeability, the permittivity,

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and the speed of light in free space, respectively. The operating frequency is f. The a_m and γ_m are the separation constants in the y direction. Matching of H_z across the two dielectric interfaces demands that

$$\frac{\tan(a_m a + \psi_m)}{a_m} = \frac{\tan \gamma_m a}{\gamma_m} \tag{7}$$

and

$$\frac{\tan(a_m b + \psi_m)}{a_m} = \frac{-\tan \gamma_m (l - b)}{\gamma_m}. \tag{8}$$

Eliminating the phase constant ψ_m, one arrives at

$$-\tan a_m(b - a) = \frac{(a_m/\gamma_m)[\tan \gamma_m(l - b) + \tan \gamma_m a]}{1 - (a_m^2/\gamma_m^2) \tan \gamma_m(l - b) \tan \gamma_m a}. \tag{9}$$

Also, a_m and γ_m satisfy

$$a_m^2 - \gamma_m^2 = (\epsilon_r - 1)k_0^2 \tag{10}$$

and

$$a_m^2 - \epsilon_r \gamma_m^2 = (\epsilon_r - 1)h_m^2. \tag{11}$$

The normalization constant Λ_m is obtained from the orthogonal relation

$$\int_0^l F_m(y) F_{m'}(y)\, dy = \delta_{mm'} \tag{12}$$

where $\delta_{mm'}$ is the Kronecker delta function.

III. THE EQUIVALENT CIRCUIT PARAMETERS OF AN ASYMMETRICALLY DIELECTRIC-SLAB-FILLED WAVEGUIDE JUNCTION

By replacing the symmetrical field-distribution function with the asymmetrical one of (4), the modal-expansion and matrix-inversion method employed in [12] is quickly adapted for the asymmetrically filled waveguide junction and one arrives at the same equivalent circuit of Fig. 1(b). This is not surprising since a central-slab-filled waveguide junction is but a special case of the asymmetrical ones. The following should be pointed out. 1) Since the two-port equivalent circuit is based on the assumption that only the dominant TE_{01} mode propagates in each waveguide, it is not valid for cases in which higher order modes may also propagate. 2) The voltage and current of the equivalent circuit are defined to be the sum of the incident and reflected E_x and the difference of the incident and reflected H_y, respectively.

The characteristic wave impedance of the asymmetrically filled waveguide is obtained from (5), where h_m is defined by (11) and a_m and γ_m are solved from a modified form of (9) and from (10), using Descartes' Rules of Signs. The modified form of (9) was employed in the computation to avoid difficulties derived from singularities. The characteristic wave impedances are further normalized to the impedance of the unloaded waveguide, Z_e, at the operating frequency and are presented as z_0 in Fig. 2 for several ϵ_r as a is changed. The same graph also represents the changes in normalized wavelength $(\lambda^{(p)}/\lambda_0)$ of the asymmetrically filled waveguide as the slab is moved across the y direction, where $\lambda^{(p)}$ and λ_0 are the waveguide wavelengths of the asymmetrically dielectric-slab-filled waveguide and the unloaded waveguide, respectively. Thus for a given slab thickness τ and slab movement a, a slab with larger ϵ_r results in a larger change of $\lambda^{(p)}$ and leads to a shorter slab. Similar results can also be obtained by increasing the thickness τ of the phase-shifter slab.

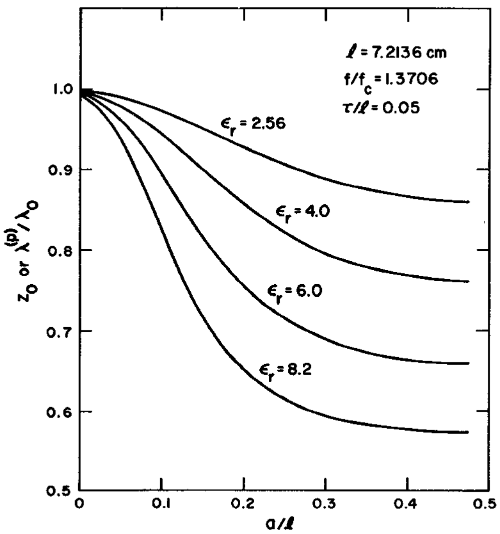

Fig. 2. Curves of normalized characteristic impedance z_0 or normalized wavelength $(\lambda^{(p)}/\lambda_0)$ for thin slabs with different ϵ_r.

Another important parameter in the design of a phase shifter is to reduce the reflection it introduces. Following the modal-expansion and matrix-inversion method [12], the series reactances X_1 and X_2 and shunt admittance B_m of an asymmetrically filled waveguide junction of Fig. 1(a) are obtained. They are normalized to Z_e and presented as $-x_1, x_2$, and b_m in Fig. 3 for slabs with $\tau/l = 0.05$ and $\tau'/l = 0.0$, and at 2.85 GHz ($f/f_c = 1.3706$, where f_c is the cutoff frequency for the TE_{01} mode). They are generally small for thin slabs. To understand these junctions, the VSWR at the unloaded waveguide ($\tau' = 0.0$) are calculated using the equivalent circuit of Fig. 1(b) and with the output of the asymmetrically filled waveguide matched. The results of the calculations with and without the junction-reactance network included are presented in curves ① and ② of Fig. 4. They show that reflections due to these junctions expressed in terms of VSWR ρ are caused primarily by the ideal transformer (i.e., the effect of discontinuity in the characteristic wave impedance across the junction). These reflections increase rapidly as ϵ_r becomes larger. Thus some impedance matching is required if material with large ϵ_r is to be

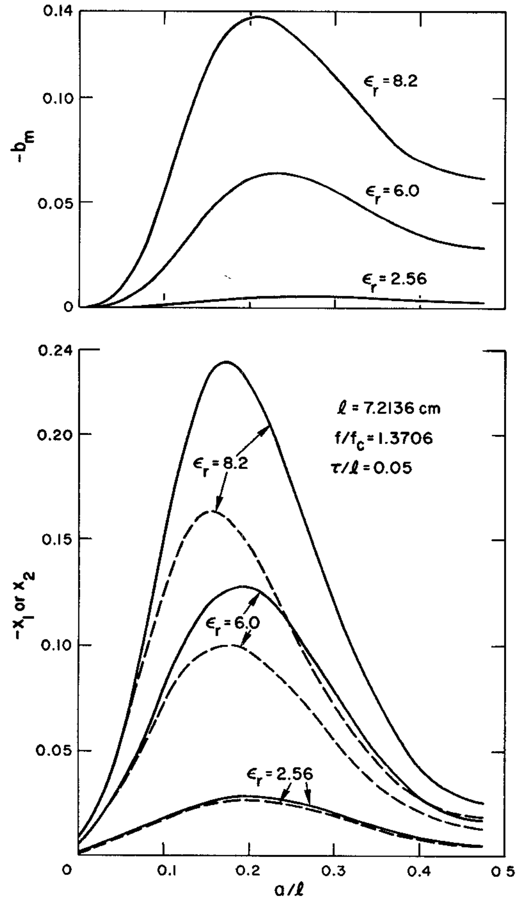

Fig. 3. Curves of normalized series impedances $-x_1$ (dashed lines) and x_2 (solid lines) and normalized shunt admittance b_m (upper graph) for junctions between an asymmetrically filled waveguide and an unloaded waveguide.

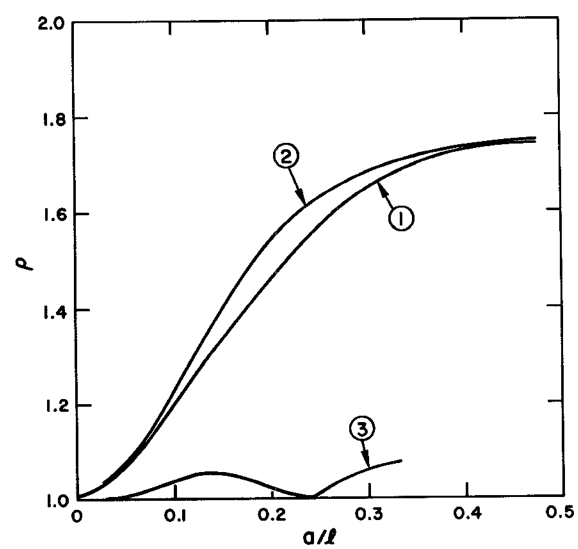

Fig. 4. Calculated VSWR ρ for junction between asymmetrically filled waveguide and unloaded waveguide. VSWR's are calculated at unloaded waveguide with asymmetrically filled waveguide matched. Curves ① and ② are calculated with and without the effect of junction-reactance network included; curve ③ is calculated by inserting a matching transformer.

employed. The inclusion of the junction-reactance network tends to reduce overall reflection.

IV. Impedance Matching Using a Single-Step Transformer with Reduced Slab Width

Borrowing from the classical transmission-line theory, the matching between two waveguides with normalized characteristic wave impedance, z_1 and z_2, is accomplished by inserting a quarter-wavelength-long waveguide section between them if the function reactances are small and can be neglected. The normalized characteristic wave impedance of the matching (transformer) section satisfies

$$z_t = (z_1 z_2)^{1/2}. \tag{13}$$

Alternatively, (13) can be written as

$$h_t = (h_1 h_2)^{1/2} \tag{14}$$

where h_1, h_2, and h_t are the propagation constants of the two original waveguides and the transformer section, respectively.

With k_0, h_t, and ϵ_r given, both the length of the transformer section d ($\equiv 0.5\pi/h_t$) and the values of a_1 and γ_1 of the TE_{01} mode in the transformer section are specified. Thus infinite sets of a and b for the transformer can be obtained. To avoid confusion, let the a and b for the transformer section be a' and b' (see Fig. 5). By introducing

$$a' = (a + p\tau) - p\tau'$$

and

$$b' = (a + p\tau) + (1 - p)\tau' \tag{15}$$

where $\tau = b - a$, $\tau' = b' - a'$, and $0 \leq p \leq 1$, we can solve the thickness τ' of the matching transformer from (9) for different p at various a. A typical set of τ'-versus-a curves is shown in Fig. 6. Since τ' changes drastically with a for the cases of $p = 0$ (i.e., a and a' line up) and $p = 1$ (i.e., b and b' line up), an intermediate value of p is preferred if a fixed transformer is to be employed.

For alumina slab ($\epsilon_r = 8.2$) operating at $f/f_c = 1.3706$ with $\tau = 0.36068$ cm ($\tau/l = 0.05$), most of the phase shift happens at $0 \leq a/l \leq 0.3325$ (see Fig. 2). One finds that the smallest variation in τ' for this range is obtained with $p \approx 0.7$. Since the junction reactances for both the transformer-to-unloaded-waveguide and the phase-shifter-to-transformer junctions are much smaller than those given in Fig. 3 for the same ϵ_r, the choice of τ' can be guided by comparing z_t^2 to z_p (normalized characteristic wave impedance for the phase-shifter section). These curves are shown in Fig. 7, which shows that the maximum value of $|z_p - z_t^2|$ over the whole range is the smallest for $\tau' \approx$ 0.18 cm. By inserting an alumina transformer of τ' = 0.180 cm and d = 3.2 cm between a semi-infinite alumina phase-shifter slab with τ = 0.3607 cm and the unloaded

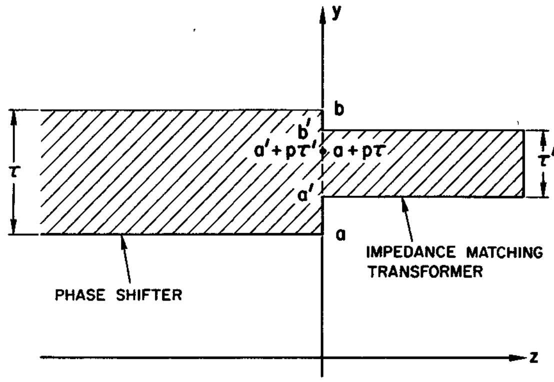

Fig. 5. Relative position between phase-shifter slab and impedance-matching transformer.

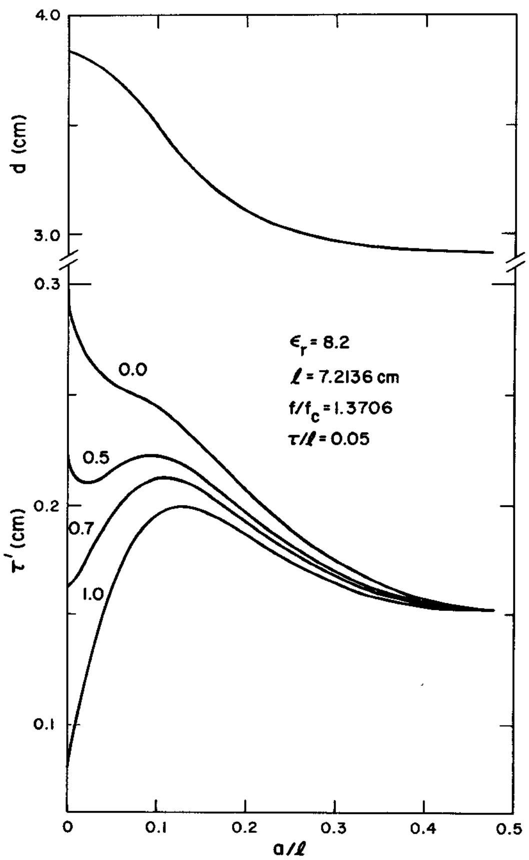

Fig. 6. Length d and thickness τ of impedance-matching transformer versus a of phase-shifter slab. Fractional numbers above each τ' curve indicate values of p for that curve.

waveguide, we again compute the VSWR ρ at the unloaded waveguide and present the results in curve ③ of Fig. 4. This shows a considerable reduction in reflection from the unmatched junction (curve ①).

For a 360° transformer-matched phase shifter, the minimum length L for the entire phase shifter is

$$L = \frac{1 - 2d[h_{\max}{}^{(t)} - h_{\min}{}^{(t)}]}{h_{\max}{}^{(p)} - h_{\min}{}^{(p)}} \tag{16}$$

where $h_{\max}{}^{(t)}, h_{\min}{}^{(t)}, h_{\max}{}^{(p)}$, and $h_{\min}{}^{(p)}$ are the maximum and minimum propagation constants of the matching-

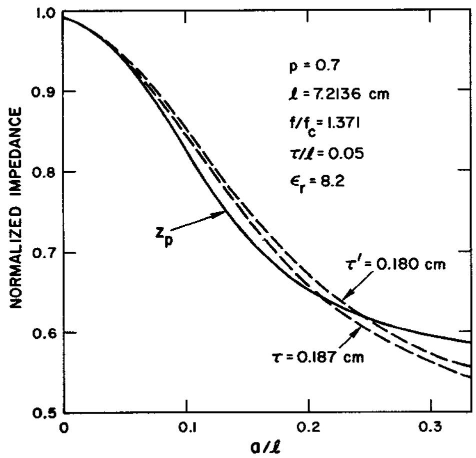

Fig. 7. Plots of z_p (solid line) and $z_t{}^2$ (dashed lines) versus a/l.

transformer and phase-shifter sections at the lowest operating frequency. The calculated VSWR ρ and differential phase shift $\Delta\theta$ for a phase shifter with $\epsilon_r = 8.2$, $\tau/l = 0.05$, $\tau'/l = 0.02495$, and $d = 3.2$ cm are presented in Figs. 8 and 9. The length of this 360° phase-shifter section is 19.885 cm long, or about 1.1 times the unloaded waveguide wavelength at the lowest operating frequency ($f = 2.7$ GHz). An alumina slab with the dimensions specified previously was fabricated to provide a comparison. Measurements of the VSWR and $\Delta\theta$ were performed at 2.85 GHz with the phase shifter terminated into a matched load and a short, respectively. The slab is supported by two small wooden posts (0.22 cm in diameter and separated from each other by 7.5 cm) through small holes from one side wall. The phase shift is produced

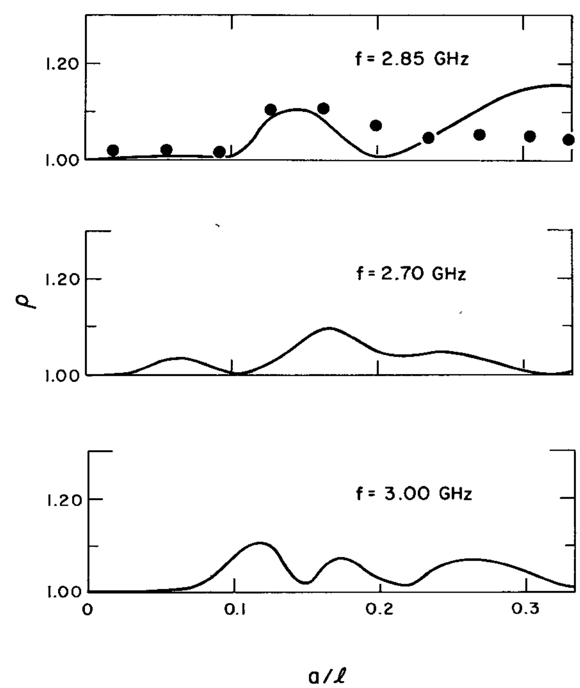

Fig. 8. Calculated curves and measured points of VSWR at input for phase shifter with $l = 7.2136$ cm, $\tau/l = 0.05$, $\tau'/l = 0.02495$, $\epsilon_r = 8.2$, $d = 3.2$. and $L = 19.885$ cm.

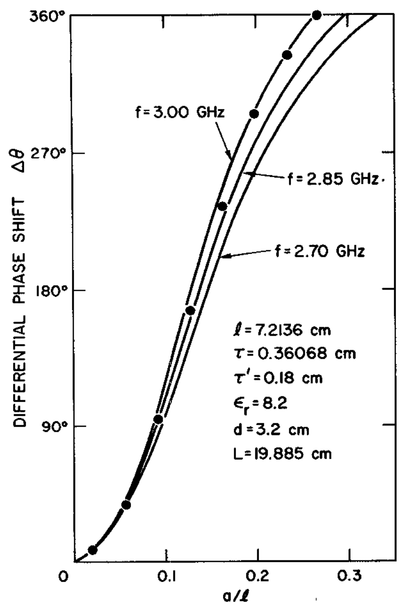

Fig. 9. Calculated curves and measured points (at $f = 2.85$ GHz) of differential phase shift $\Delta\theta$ for the same slab-filled phase shifter given in Fig. 8.

by pushing (or pulling) these posts into (or out of) the waveguide. Results of these measurements were presented as "dots" in both Figs. 8 and 9. The results are in good agreement with the calculations. The small deviation at larger a/l is due chiefly to the effect of the supporting posts.

V. CONCLUSION

The equivalent circuit for the junction between two asymmetrically filled waveguides and the parameters of this circuit are obtained and studied. This enables the conversion of a complicated field problem to a simpler circuit problem, and leads to a better understanding of the problem of impedance matching for slab-filled phase shifters. With these basic understandings, a wide-band impedance matching using a multisectional transformer can be easily developed.

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